MBA-003-001408 Seat No. ______

B. Sc. (Sem. IV) (CBCS) Examination

March / April - 2018

Mathematics : Paper - 401 (A)

(Advanced Calculus & Linear Algebra)

Faculty Code : 003
Subject Code : 001408

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following : **20**

(1) $y = p\cos\theta,\ \ z = p\sin\theta$ then $\frac{\partial p}{\partial y} =$ ________

(2) If $u = cx^2$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ________

(3) $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} =$ ________

(4) If error of 3% in E and in R are made then the percentage error in $p = \frac{E^2}{R}$ is = ?

(5) Maximum value of $x^2 + y^2 + 6x + 14$ is = ?

(6) If A, B, C are angles of a triangle ABC then the maximum value of $\cos A \cos B \cos C$ is = ?

(7) In usual notations $div\,\overline{r} =$ ______

(8) If $\Phi = x^2 + y^2 + z^2$ then $grad\ \phi =$ _______

(9) $\overline{f} = 2x^2y + 2y^3 + 3z^2$ then $\nabla^2\ \overline{f} =$ _________

(10) $div\,(3\,z)$ =

(11) Relation between Cartesian co-ordinate and cylindrical co-ordinate is = ?

(12) $\int_{-a}^{a}\int_{0}^{x} dydx =$ _______

(13) According to Stokes theorem $\int_{C} \overline{u} \cdot \overline{dr} =$ _____

(14) According to divergence theorem

$\iint_{S} Ldydz + Mdzdx + Ndxdy =$ ______

(15) $\overline{\big)}\frac{1}{3}\frac{3}{4} =$ _______

(16) $\int_{0}^{\infty} e^{-x} x^{3} dx =$ _______

(17) $\overline{\big)}2n =$ _____

(18) If $u = (1, 2, 3)$ and $v = (1, 0, 1)$ are vectors of R^3 then $u \cdot v =$ ______

(19) If $u \in R^3$, R^3 is an Euclidean space, $u = (2, 1, -1)$ then $\|u\| =$ ________

(20) $\overline{\overline{F}}$ is solenoidal if ?

2 (a) Answer any **three** : **6**

(1) If $x^3 + y^3 + z^3 - 3xyz = 0$ then find $\frac{\partial z}{\partial x}$.

(2) If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

(3) Expand $e^x \sin y$ in power of x and y.

(4) If $u = \frac{yz}{x}, v = \frac{xz}{y}, w = \frac{yx}{z}$ then show that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$$

(5) If $\overline{f}$ is solenoidal then find a.

$$\overline{f} = (ax + 3y + 4z)\,\overline{i} + (x - 2y + 3z)\,\overline{j} + (3x + 2y - z)\,\overline{k}$$

(6) If $\overline{f} = x^2 y\,\overline{i} - 2xz\,\overline{j} + 2yz\,\overline{k}$ then find $curl\ \overline{f}$.

(b) Average any **three** : **9**

(1) Using the definition of partial derivatives find f_x

and f_y $f(x, y) = \begin{cases} \frac{x(x^2 - y^2)}{x^2 + y^2} : (x, y) \neq (0, 0) \\ 0, (x, y) = (0, 0) \end{cases}$

(2) If $u = f(z)$ and $z^2 = x^2 + y^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(z) + \frac{1}{z} f'(z)$$

(3) Expand $x^2y+3y-2$ in power of $x-2$ and $y-3$.

(4) Find maximum and minimum x^3+y^3-3xy

(5) If $\overline{f}$ is irrotational then find a, b, c

$$\overline{f}=(2x+3y+az)\,\overline{i}+(bx+2y+3z)\,\overline{j}+(2x+cy+3z)\,\overline{k}$$

(6) If $r=|\overline{r}|$ where $\overline{r}+x\,\overline{i}+y\,\overline{j}+z\,\overline{k}$ then prove that

$$\nabla f(\overline{r})=f'(r)=\frac{\overline{r}}{r}$$

(c) Answer any **two** : **10**

(1) If $v=\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ and $u=\sin^{-1}v$ then show that

$$x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+2xy\frac{\partial^2 u}{\partial x\partial y}=\frac{1}{4}\tan u\left(\tan^2 u-1\right).$$

(2) Divide 120 into three roots such that the sum numbers is maximum.

(3) Prove that $\dfrac{1}{r}$ satisfies the Laplace equation.

(4) If $u=f(y-z, z-x, x-y)$ then show that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

(5) Find the shortest distance from origin to the surface $xyz^2=2$.

3 (a) Answer any **three** : **6**

(1) $\iint_R x \sin(x+y)\, dxdy$ where $R = [0, \pi \,;\, 0, \pi/2]$

(2) $\iiint_R xydxdydz$ where R is a cube, $0 \le x, y, z \le 1$.

(3) Find $\int_{(0,0)}^{(2,2)} y^2 dx$

(4) Find $\int_{(0,0)}^{(1,1)} xds$

(5) Let R^3 have Euclidean inner product and $u = (-1, 5, 2)$, $v = (2, 4, -9)$. Then find angle between u and v.

(6) If u, v and w are vectors in an inner product space v and α be scalar then prove

(a) $(u+v) \cdot w = u \cdot w + v \cdot w$

(b) $u \cdot (\alpha v) = \overline{\alpha}(u \cdot v)$

(b) Answer any **three** : **9**

(1) Evaluate $\iiint_R x^2 dxdydz$ where R is a cube,

$0 \le x, y, z \le 1$.

(2) $\int_C V_n ds$ where C is $x^2 + y^2 = 1$ and

$\overline{V} = \left(x^2 + y^2\right)\overline{i} - 2xy\,\overline{j}$

(3) Prove that $\int_{0}^{\frac{\pi}{2}} \sin^{m}\theta \cos^{n}\theta d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$

(4) Prove that $p\beta(p, q+1) = q\beta(p+1, q)$

(5) State the prove Triangular Inequality.

(6) Using Stoke's theorem find

$\int_{C} 2xy^{2}zdx + 2x^{2}yzdy + \left(x^{2}y^{2} - z\right)dz$ where C is

$x^{2} + y^{2} + z^{2} = a^{2}$ boundary of hemisphere.

(c) Answer any **two** : **10**

(1) Prove $\iint_{R} (1-x-y)^{3} x^{1/2} y^{1/2} dxdy = \frac{\pi}{480}$ where R is triangular whose vertices are $(0, 0), (0, 1)$ and $(1, 0)$.

(2) State and prove Green's Theorem.

(3) Prove $\int_{0}^{1} \frac{x^{5}}{\sqrt{1-x^{4}}} dx = \frac{\pi}{8}$

(4) Let R^{3} have the Euclidean inner product > Transform the basis $S = \{(1, 0, 0), (3, 7, -2), (0, 4, 1)\}$ into an orthogonal basis using Gram Schmidt Process.

(5) Find $\iint_S x^2 dydz + y^2 dzdx + 2z dxdy$ where

$S : 0 \le x, y, z \le 1$ a solid surface
